Using Differentiation

Exercise 11B

1 Find an expression for **i** the velocity and **ii** the acceleration of a particle given that the displacement is given by:

a $s = 4t^4 - \frac{1}{t}$ **b** $x = \frac{2}{3}t^3 + \frac{1}{t^2}$ **c** $s = (3t^2 - 1)(2t + 5)$ **d** $x = \frac{3t^4 - 2t^3 + 5}{2t}$

2 A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 2t^3 - 8t$. Find:

a the velocity of the particle when $t = 3$ **b** the acceleration of the particle when $t = 2$.

(P) **3** A particle P is moving on the x-axis. At time t seconds (where $t \geqslant 0$) , the velocity of P is v m s^{-1} in the direction of x increasing, where $v = 12 - t - t^2$.

Find the acceleration of P when P is instantaneously at rest.

(P) **4** A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 4t^3 - 39t^2 + 120t$.

Find the distance between the two points where P is instantaneously at rest.

(E/P) **5** A particle P moves in a straight line. At time t seconds the acceleration of P is a m s^{-2} and the velocity v m s^{-1} is given by $v = kt - 3t^2$, where k is a constant.

The initial acceleration of P is 4 m s^{-2}.

a Find the value of k. **(3 marks)**

b Using the value of k found in part **a**, find the acceleration when P is instantaneously at rest. **(3 marks)**

(E/P) **6** The print head on a printer moves such that its displacement s cm from the side of the printer at time t seconds is given by:

$$\tfrac{1}{4}(4t^3 - 15t^2 + 12t + 30),\ 0 \leqslant t \leqslant 3$$

Find the distance between the points when the print head is instantaneously at rest, in cm to 1 decimal place. **(6 marks)**

Using Integration

Exercise 11D

1 A particle is moving in a straight line. Given that $s = 0$ when $t = 0$, find an expression for the displacement of the particle if the velocity is given by:

a $v = 3t^2 - 1$ **b** $v = 2t^3 - \frac{3t^2}{2}$ **c** $v = 2\sqrt{t} + 4t^2$

2 A particle is moving in a straight line. Given that $v = 0$ when $t = 0$, find an expression for the velocity of the particle if the acceleration is given by:

a $a = 8t - 2t^2$ **b** $a = 6 + \frac{t^2}{3}$

3 A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(8 + 2t - 3t^2)$ m s^{-1} in the direction of x increasing. At time $t = 0$, P is at the point where $x = 4$. Find the distance of P from O when $t = 1$.

4 A particle P is moving on the x-axis. At time t seconds, the acceleration of P is $(16 - 2t)\,\text{m s}^{-2}$ in the direction of x increasing. The velocity of P at time t seconds is $v\,\text{m s}^{-1}$. When $t = 0$, $v = 6$ and when $t = 3$, $x = 75$. Find:

a v in terms of t **b** the value of x when $t = 0$.

(P) **5** A particle is moving in a straight line. At time t seconds, its velocity, $v\,\text{m s}^{-1}$, is given by $v = 6t^2 - 51t + 90$. When $t = 0$ the displacement is 0. Find the distance between the two points where P is instantaneously at rest.

(P) **6** At time t seconds, where $t \geqslant 0$, the velocity $v\,\text{m s}^{-1}$ of a particle moving in a straight line is given by $v = 12 + t - 6t^2$. When $t = 0$, P is at a point O on the line. Find the distance of P from O when $v = 0$.

(P) **7** A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(4t - t^2)\,\text{m s}^{-1}$ in the direction of x increasing. At time $t = 0$, P is at the origin O. Find:

a the value of x at the instant when $t > 0$ and P is at rest

b the total distance moved by P in the interval $0 \leqslant t \leqslant 5$.

Problem-solving

You will need to consider the motion when v is positive and negative separately.

(P) **8** A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(6t^2 - 26t + 15)\,\text{m s}^{-1}$ in the direction of x increasing. At time $t = 0$, P is at the origin O. In the subsequent motion P passes through O twice. Find the two non-zero values of t when P passes through O.

(P) **9** A particle P moves along the x-axis. At time t seconds (where $t \geqslant 0$) the velocity of P is $(3t^2 - 12t + 5)\,\text{m s}^{-1}$ in the direction of x increasing. When $t = 0$, P is at the origin O. Find:

a the values of t when P is again at O

b the distance travelled by P in the interval $2 \leqslant t \leqslant 3$.

(P) **10** A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geqslant 0$, is $(4t - 3)\,\text{m s}^{-2}$ in the positive x-direction. When $t = 0$, the velocity of P is $4\,\text{m s}^{-1}$ in the positive x-direction. When $t = T$ $(T \neq 0)$, the velocity of P is $4\,\text{m s}^{-1}$ in the positive x-direction. Find the value of T. **(6 marks)**

(E) **11** A particle P travels in a straight line such that its acceleration at time t seconds is $(t - 3)\,\text{m s}^{-2}$. The velocity of P at time t seconds is $v\,\text{m s}^{-1}$. When $t = 0$, $v = 4$. Find:

a v in terms of t **(4 marks)**

b the values of t when P is instantaneously at rest **(3 marks)**

c the distance between the two points at which P is instantaneously at rest. **(4 marks)**

(E/P) **12** A particle travels in a straight line such that its acceleration, $a\,\text{m s}^{-2}$, at time t seconds is given by $a = 6t + 2$. When $t = 2$ seconds, the displacement, s, is 10 metres and when $t = 3$ seconds the displacement is 38 metres. Find:

a the displacement when $t = 4$ seconds **(6 marks)**

b the velocity when $t = 4$ seconds. **(2 marks)**

Problem-solving

You need to use integration to find expressions for the velocity and displacement then substitute in the given values. Use simultaneous equations to find the values of the constants of integration.